

For 0 $\rightarrow y(t) = A p_1(t) + n(t)$

For 1 $\rightarrow y(t) = A p_2(t) + n(t)$

optimal Receive Filter $h(t)$
 $h(t) = p_1(T-t) - p_2(T-t)$
Maximizes output SNR

Consider information bit 0.

$$y(t) = A p_1(t) + n(t).$$

Receive filter with $h(t) = p_1(T-t) - p_2(T-t)$
Followed by sampling at $t = T$

$$r(T) = \int_{-\infty}^{\infty} y(\tau) h(T-\tau) d\tau$$

$$= \int_{-\infty}^{\infty} (A p_1(\tau) + n(\tau)) \times (p_1(\tau) - p_2(\tau)) d\tau$$

$$= \int_{-\infty}^{\infty} A p_1(\tau) (p_1(\tau) - p_2(\tau)) d\tau$$

Signal component

$$+ \underbrace{\int_{-\infty}^{\infty} n(\tau) \cdot (P_1(\tau) - P_2(\tau)) d\tau}_{\text{Noise component}}$$

$$\begin{aligned} \text{Signal} &= \int_{-\infty}^{\infty} A P_1(\tau) (P_1(\tau) - P_2(\tau)) d\tau \\ &= A \underbrace{\int_{-\infty}^{\infty} P_1^2(\tau) d\tau}_{\mathcal{E}_p = 1} - A \underbrace{\int_{-\infty}^{\infty} P_1(\tau) P_2(\tau) d\tau}_{\substack{\text{inner product} \\ \text{of } P_1(\tau), P_2(\tau) \\ = 0.}} \end{aligned}$$

$$= A \cdot \mathcal{E}_p = A \quad \leftarrow \text{Signal component.}$$

$$\tilde{n} = \int_{-\infty}^{\infty} n(\tau) (p_1(\tau) - p_2(\tau)) d\tau$$

Gaussian, mean = 0.

$$\text{var} = \frac{N_0}{2} \int_{-\infty}^{\infty} |h(T-\tau)|^2 d\tau$$

$$= \frac{N_0}{2} \int_{-\infty}^{\infty} (p_1(\tau) - p_2(\tau))^2 d\tau$$

$$= \frac{N_0}{2} \int_{-\infty}^{\infty} (p_1^2(\tau) + p_2^2(\tau) - 2p_1(\tau)p_2(\tau)) d\tau$$

$$\begin{aligned}
 &= \underbrace{\frac{N_0}{2} \int_{-\infty}^{\infty} P_1^2(\tau) d\tau}_{E_P=1} + \underbrace{\frac{N_0}{2} \int_{-\infty}^{\infty} P_2^2(\tau) d\tau}_{E_P=1} \\
 &\quad + \underbrace{\frac{N_0}{2} \int_{-\infty}^{\infty} P_1(\tau) P_2(\tau) d\tau}_{\text{inner product} = 0}
 \end{aligned}$$

$$= \frac{N_0}{2} + \frac{N_0}{2} + 0 = N_0$$

Noise power after sampling.

$$\begin{aligned}
 r(T) &= A \mathcal{E}_p + \tilde{n} \\
 &= A + \tilde{n}
 \end{aligned}$$

Gaussian
mean = 0
var = N_0 .

Corresponding to bit 1

$$y(t) = A p_2(t) + n(t).$$

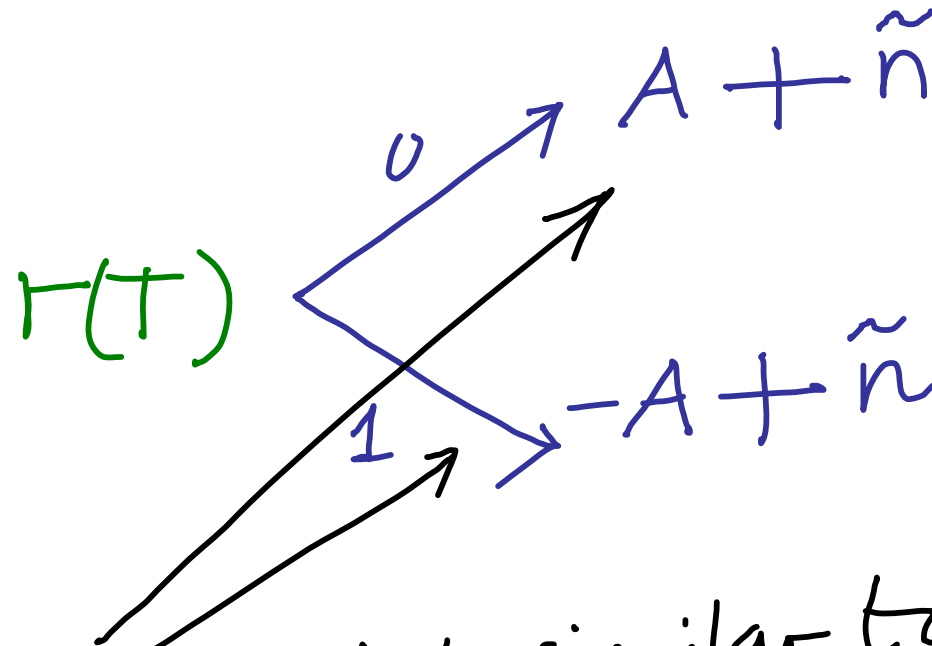
Matched Filter

$$h(t) = p_1(T-t) - p_2(T-t)$$

$$r(T) = -A \mathcal{E}_p + \tilde{n}$$

$$= -A + \tilde{n}$$

Gaussian
mean = 0
var = N_0 .



Model is similar to BPSK
Except $\sigma^2 = N_0$.

By symmetry, optimal Decision rule is,

$$\begin{aligned} r(T) \geq 0 &\Rightarrow \text{Decide } a_0 = 0. \\ r(T) < 0 &\Rightarrow \text{Decide } a_0 = 1 \end{aligned}$$

Optimal Threshold based Decision rule for Frequency Shift Keying (FSK).

$$\text{BER} = Q\left(\frac{A}{\sigma}\right)$$

Bit Error Rate
Probability of Bit Error

$$= Q\left(\frac{A}{\sqrt{N_0}}\right)$$

$$= Q\left(\sqrt{\frac{A^2}{N_0}}\right)$$

Substitute $A = \sqrt{E_b}$
 $E_b = \text{energy/bit}$

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

BER for Frequency Shift Keying (FSK).

$$\text{BER. BPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$\text{ASK, FSK} = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

— BER of ASK = FSK for equal average energy/bit E_b .

— BER of ASK, FSK $>$ BER of BPSK

Because $Q(\cdot)$ Function is a decreasing function

— ASK, FSK are 3dB inefficient
in comparison to BPSK

For same BER, ASK/FSK
need 3dB more power.